Studies on the structure of Gothic Cathedrals

Pere Roca
Universitat Politècnica de Catalunya, Department of Construction Engineering, Barcelona, Spain

ABSTRACT: The study of three Gothic Cathedrals is presented with a discussion on the results obtained with regard to their structural features and present condition. The paper focuses on the significant difficulties that the analysts may encounter in the attempt of evaluating historical structures. These difficulties stem from the importance of historical facts (historical genesis, construction process, and possible natural or anthropic actions affecting the construction throughout its life time) and the need to integrate them in the analysis. The case studies of Tarazona, Barcelona and Mallorca Cathedrals are presented. Particular attention is given to Mallorca Cathedral because of the uniqueness and audacity of its structural design and because of the challenges that the analyst must face to conclude about its actual condition and long-tem stability.

1 CONSIDERATIONS ON THE ANALYSIS OF ANCIENT CONSTRUCTIONS

One of the main elements of the European architectural heritage is found in the significant number of Gothic churches and cathedrals built during the last period of the Middle Age. Preserving these constructions requires a certain understanding of their structural features and stability condition; this, in turn, requires a certain knowledge of their material, construction and resisting nature.

However, the attempt to understand a complex ancient construction, such as a Gothic cathedral, based on modern techniques and concepts, encounters important difficulties stemming from both practical and theoretical causes.

Among the difficulties of practical character is the virtual impossibility of obtaining a detailed knowledge of the properties of the materials, construction details and internal composition. The materials -masonry or rubble infill, mortar, stone blocks-, are usually very heterogeneous with largely variable mechanical properties. Given the historical and artistic value of these buildings, it is not advisable to carry out a very exhaustive sampling because of the damage it may produce in the existent fabrics. The information generated by non-destructive or quasi non-destructive methods (flat-jack test, electromagnetic tomography, endoscopy…) is not easy to interpret and requires a very accurate previous calibration; besides, an extensive use of such methods requires a significant financial investment which is only justified in the case of very important buildings. Without an exhaustive information on materials and geometry (both external and internal), carrying out a modern analysis in strict sense turns out virtually impossible.

Among the difficulties of more theoretical or conceptual character is the fact that the study of a construction of these characteristics can not be undertaken without considering the most relevant historical facts experienced throughout its life-span. The historical conditions linked to

the conception, construction and historical vicissitudes are to be considered and, to some extend, integrated in the analysis.

Our more conventional, modern techniques of analysis contemplate the structural fact as a static reality, as a fact born instantly and remaining uniform in the time domain. Only some modern methods of analysis, mostly developed for concrete structures, can undertake realistic time-dependent analysis in the form of a long-term, or also sequential-evolutionary analysis, with the objective of simulating the different time-dependent phenomena related to the material (creep, shrinkage, ageing...) as well as the influence of the constructive process in the final state of stresses. In particular, the sequential-evolutionary analysis constitutes a very useful tool for analysing complex constructive processes or delicate strengthening operations. However, even the time-dependent or sequential analyses become insufficient to understand the reality of a building whose construction was prolonged during decades or centuries and which has eventually experienced earthquakes, hurricane winds, foundation problems, the effect of countless thermal cycles, water filtering, and many other possible actions.

Anthropic actions, due to continuous use or associated to inadequate maintenance or repair, are not to be disregarded among all the long-time possible causes of structural and material alteration.

As opposed to modern buildings, an appropriate characterisation of the materials or construction details (including internal composition) will not be available or possible in the case of many ancient constructions. However, another source of very valuable information is to be found in the history of the building itself. History can be understood as an experiment in
true geometrical and time scale, and should be considered by the analyst as a precious source of evidence on the structural features of the building.

The difficulty in understanding history as a source of evidence lies in the adequate interpretation of the historical facts. On the one hand, the single evidence that a building is standing offers an empirical hint of its viable stability; however, the actual meaning of this fact can only be completely interpreted in the light of a sound structural analysis. In the case of a historical construction, history must be understood as the basic element of study and the main source of evidence. The experimental and numeric analyses, however necessary, provide only an auxiliary tool by means of which the adequate interpretation of the historical evidence becomes more objective.

Because of the features of ancient buildings and the importance of history, our available tools of analysis, either conventional or advanced, become insufficient, when not completely inappropriate, for the assessment of such constructions. However, the engineer has yet at disposal the most powerful and general of the tools, namely the scientific method.

In short, the structural analysis allows the modelling of the hypotheses whose formulation constitutes the starting point of the scientific method. The hypotheses, as required by the scientific method, should be calibrated in an empirical way. The empirical confirmation can come from history - again, as an experiment at real scale and time, - from new, non-destructive experiments and from medium or large-term monitoring.

2 ON THE CONCEPTION OF THE STRUCTURE OF GOTHIC CATHEDRALS

In contrast with our difficulties to understand, or at least to approach the complexity of a Gothic cathedral, the capacity of the medieval master builders to conceive such constructions without the help of any method or criteria based on rational mechanics, turns out admirable.

Nowadays, the assessment of such structures is carried out by means of powerful, up-to-date tools of analysis. Or, alternately, by means of ultimate analysis. According to many experts, ultimate analysis is still (and will be) the most appropriate tool for the study of this type of constructions, no matter the capabilities of the modern FEM-based applications. However, even ultimate analysis has a relatively modern origin going back to the works by Coulomb and other specialists from the 17th and 18th centuries.

Figure 1: Nave of Mallorca Cathedral

Figure 2: Nave of Girona Cathedral

Many Gothic cathedrals present a highly optimised structure with regard to the resisting needs caused by the gravitational action. By speaking of high optimisation, we refer to the fact that the master builders knew how to extract the biggest profit from the material available to them. In some cases, structural components were built with reduced sections and great slendernes, to the extend of reaching and even overcoming reasonable limits according to our modern understanding. In can be said that, at least in a number of cases, they used the minimum quantity of material granting structural stability.

Mallorca Cathedral, described in section 6, constitutes a prominent example of the previous ideas. The central nave, 44 m high at the vault keystone and having 17.8 m of span, is sustained on piers with a diameter of 1.6 or 1.7 m (Fig. 1), characterised by an slenderness (defined as ratio between the free height to the diameter) of 14.1. These proportions would seem audacious in a modern reinforced concrete column; nevertheless, they correspond to a masonry construction, that is to say, to a construction composed

As mentioned, one of the difficulties posed to the analysts by ancient constructions is found in the understanding of the historical concepts and processes originally generating and governing their geometry and structural arrangement. This is so, particularly, in the case of Gothic construction because of the poor knowledge we have on the Medieval science of construction.

entirely of a brittle material which can resist no tension and only moderate compression.

The discussions held by contemporary experts a propose of the construction of some cathedrals, documented historically - like in the case of the unique nave of Girona Cathedral or Milano Cathedral – do not provide any significant hint on the technical reasoning of the medieval master builders. These discussions seem to illustrate, indeed, the absence of fully objective arguments in relation to the structural design and the resisting possibilities of the structural components. However, some experts attribute the absence of such arguments to the zeal of the master builders in protecting their own knowledge.

It is remarkable that, in relation to the construction of the unique nave of Girona Cathedral (Fig. 2), some experts of the time, invited by the Chapter in the year 1416, agreed to consider that the construction of a cross-vault with a span of more than 23 m was possible and appropriate; they agreed to validate the capacity of the existing walls and foundations to receive the weight and the lateral thrust of the large vaults of the unique nave. However, some of the experts pointed out that the stability might not be granted in the event of an earthquake or hurricane wind. Although, as mentioned, no rationale or technical arguments were exhibited to sustain the opinions of the experts, yet it is very patent that they had the knowledge that allowed them to conclude on delicate structural facts. (A more detailed description of the discussions happened during the meeting in 1416 can be found in Huerta 1998).

The first modern attempts to rationally understand the structure of Gothic cathedrals, coming from the 19th c., produced a very enthusiastic acceptance of the rationality and sufficiency of their structural arrangement. Viollet-le-Duc writings invited to regard gothic architecture as a fully rational system strictly based on equilibrium. Oppositely, modern experts have pointed to significant inconsistencies or arbitrarities in the structural design of many Cathedrals which can only be understood because of the limitations of the knowledge of the builders or because of architectural or liturgical priorities.

But the efficiency of the Gothic construction, if exists, doesn't come so much from a detailed and conscientious rational design, as of the fact of being sheerly based on a powerful and rational principle, namely thrust equilibrium. In spite of possible lack of sound rationality, Gothic structure is intensively based on the equilibrium of thrusts; that is to say, on the transmission of the thrusts generated by arches and vaults, until reaching the foundation, through secondary arches, flying arches and buttresses. Being governed by this principle, the construction acquires large efficiency until becoming an actual stone skeleton. Also, being governed by this principle, the problem of equilibrium becomes mostly a geometrical problem not strongly dependent with the strength properties of the materials.

As it can be inferred from the principles of ultimate analysis (see in Heyman 1995, and Huerta 1996), usually, the compression strength of the stone or the fabric has scarce influence

on the global strength of a structure based on equilibrium of thrusts. The key condition is of geometric nature: the thrust lines must not exceed the sections of the structural components and must become tangent to its surface in a number of points less than the number of hinges needed to produce a ductile mechanism. Because of that, the design of a construction of this kind may be based on rules of elementary or schematic character, such as those that relate certain geometric proportions of the arch to the base of the buttress that sustains it (such as the "rule of Blondel", see in Hureta 1996).

3 STUDY OF THREE GOTHIC CATHEDRALS

Spain's architectural heritage includes a large number of Gothic Cathedrals with very different architectural and structural patterns or styles. Some of them -the earlier ones, including some outstanding examples, as Burgos or León Cathedrals- were fashioned within the classical French High Gothic architecture of 13th c. Other examples, as those built in the Mediterranean territories of the Kingdom of Aragón during 14th and 15th c., correspond to a more evolved architecture showing characteristic architectural features and structural innovations.

The analyses here referred to correspond to the study of the naves of Tarazona, Barcelona and Mallorca Cathedrals. The study of Mallorca Cathedral here presented constitutes only a preliminary step towards a better understanding of the structure and its actual present condition, while a more comprehensive assessment is intended for the near future (Gonzalez and Roca 2000).

The study of the structures subject to dead load was carried out by gradually increasing the applied load until reaching its actual value, and then continuing to marginally increase it until causing the failure of the system. The failure resulted in all the studies here reported because of the development of a ductile ultimate mechanisms characterised by a certain distribution of plastic hinges. Similarly, the study of the effect of differential settlements between the piers consisted of increasing the value of the settlement until causing severe damage, and beyond, until simulating the failure of the construction.

Two methods of analysis, briefly described in Appendix 1, were used to carry out the studies. First, the Generalised Matrix Formulation for Masonry Frame Constructions (GMF), developed by Roca and Molins (1998); second, two FEM-based continuous damage models, both developed respectively by Cervera et al. (1998) and Oñate et al. (1998). Although both FEM-continuous damage models were developed, in principle, for concrete structures, they are also useful for the analysis of masonry constructions because they comprehend the most significant features of the material, such as the brittleness in tension and the yielding of the material and eventual crushing in compression. Besides, the formulations allow for large concentration of damage at certain locations as is typically observed in masonry constructions.

The mechanical properties considered for the analyses were decided based on experience available for materials similar to those existing in the buildings. The values initially assumed for the compressive strength are 6.0 MPa and 8.0 MPa for the stone masonries of Tarazona and Barcelona cathedrals respectively. The effect of possible variations with respect to the assumed values was accounted for by means of a sensitivity-analysis, as is described below.

4. TARAZONA CATHEDRAL

Tarazona Cathedral was begun to be built over the remains of a Romanesque church by 1235. The more ancient parts are the choir and transept, built during 13th; the nave and the original cimborio were built during 14th c. The second, present cimborio was erected later, during 16th c.

The dimensions of the building are rather moderate: the span of the central nave is 7.3 m. and the highness at the keystone of the vaults is about 16.5m.

Figure 3: Tarazona Cathedral. Cimborio and clerestory arches propped on steel frame and masonry diaphragms; piers encircled with timber and steel confinement.
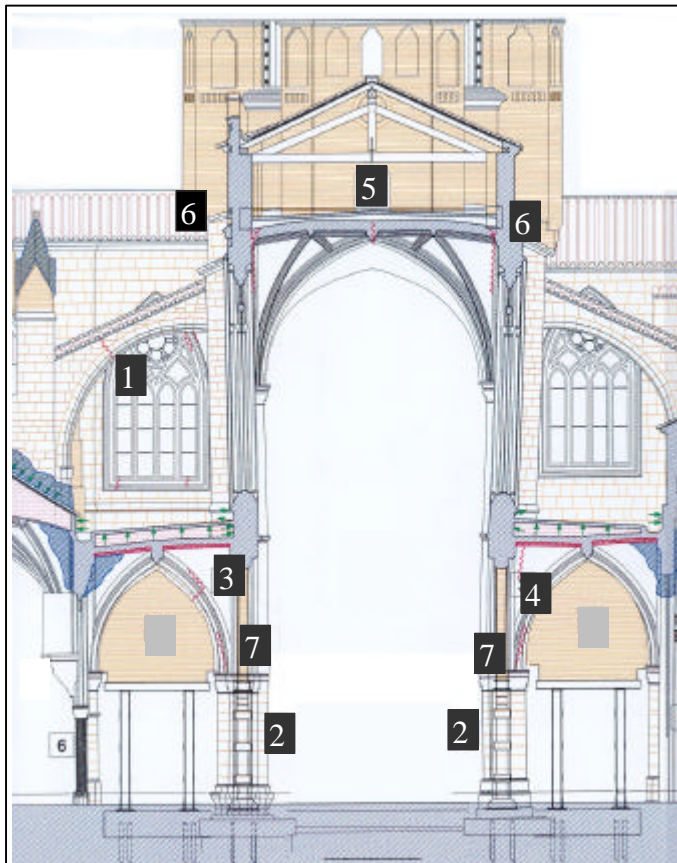

Figure 4: Tarazona Cathedral. Distribution of lesions:

    (1) Crack at flying arch
    (2) Deterioration of stone at piers
    (3) Cracks at aisle arch
    (4) Cracks at aisle vault (back
    (5) Crack at nave vault (key)
    (6) Crack at nave vault (backing)
    (7) Crushing and cracking at the springings of the arches of the aisle vaults

Figure 5: Tarazona Cathedral. Lesions related to excessive compression at the springings of the aisle arches.

Due to the material and structural deterioration of the building, a large part of it, including the cimborio and the clerestory walls, was propped on steel frames some decades ago, and the cathedral closed to public for years (Figs 3, 4). A restoration programme, including repair and strengthening operations, is now being carried out with the aim to eliminate the propping system.

Different events have contributed to deteriorate the construction. First, chemical attack has produced significant degradation of the stone of the piers to the extend of making it advisable to confine some of them with timber and steel auxiliary elements.

Second, some historical actions have caused a very significant alteration of the equilibrium of the building. Three important actions can be mentioned: (1) the construction of a heavy cimborio during the 16th c; (2) during the 16th c. part the section of the piers of the nave was removed to make space for a timber choir, causing an increase of the stresses and deformations experienced by them; (3) during 1960-1962, the flying arches were dismantled and then reconstructed as part of the activities included in a restoration.

Today, the limestone original piers and arches of the nave (Fig. 4) show damage of mechanical origin such as cracking and crushed material in some points. The main lesions observed in the nave are: (1) cracks at about mid-span of the flying arches; (2) degradation of the stone in the base of the piers; (3) in some bays, cracks developed in the transverse ribs of the arches, close to the inner springings; (4) vertical cracks separating the masonry backing of the aisle vaults from the walls; (5) a large crack at the key of the main transverse arches.; (6) vertical cracks separating the backing of the nave vaults from the clerestory walls; and (7) damage in the nervatures due to excessive compression at the inner springings of the aisles (Fig. 5).

The analysis carried out by the numerical techniques allows a correlation between the effects introduced by the historical alterations and existing lesions (Fig. 6). Thus, cracks (1), (5), (7) seem associated to the initial condition; reducing the section of the piers produced a certain extension of the zones affected by excessive tension or compression stresses; and removing the flying arches may have motivated cracks (3) and (5).

The settlement of the pier with respect to the buttress may also have collaborated in generating some of the cracks, as the one affecting the flying arc (1) in special.
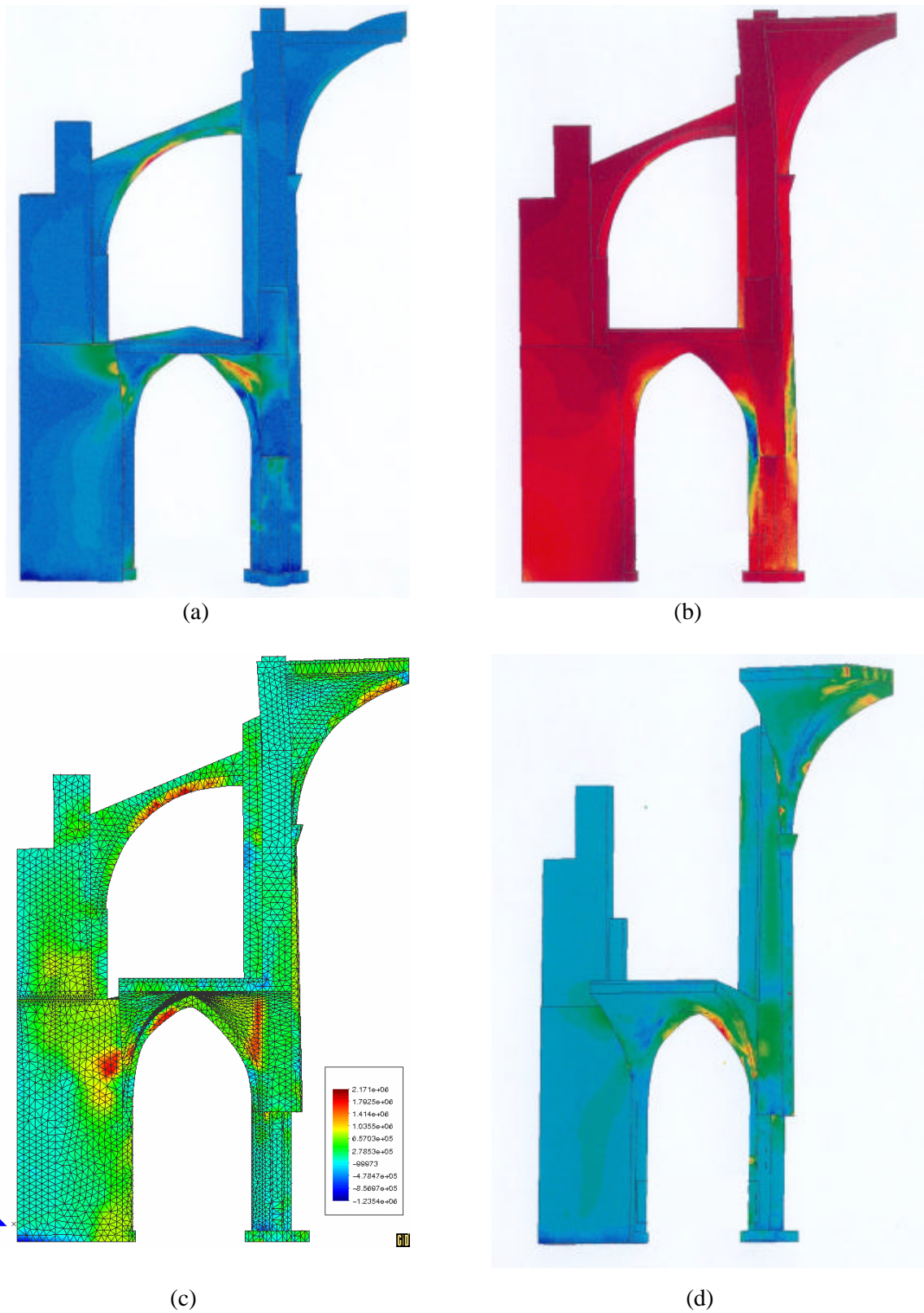
(a)

(b)

(c)

(d)

Figure 6:   Numerical model prepared for the analysis of the nave of Tarazona Cathedral by means of FEM-Continuous damage model. Distribution of maximum tension stresses (a) and maximum compression stresses (b) in the initial configuration. Distribution of tension stresses after the reduction of the section of the pier (c) and once the flying arch is removed (d).

Remarkably, the analysis predicts a rather fair, almost deficient, condition of equilibrium even for the initial, intact configuration. According to GMF, failure would occur at 110% the dead loading, while the FEM damage model predicts the failure for 130% the dead load. Given the existing damage and given the very small marginal capacity predicted, the need to prop the construction until the implementation of some strengthening measures seems justified. The effects of anthropic actions and other historical causes may have been so severe because of the initial precarious condition and the resulting sensitivity to any possible alteration.

The parametrical studies showed that a possible increase of the compressive strength of the materials would not provide a significant increase of the ultimate capacity. However, a moderate decrease of the compressive strength leads, according to the study, to a significant reduction of the total dead load that can be resisted by the system.

The study of te cimborio of Tarazona Cathedral has been already presented (Roca and Molins 2000).

5 BARCELONA CATHEDRAL

Construction of the naves of Barcelona Cathedral (Fig. 7) was begun in 1298 and lasted for more than a century. As usual, the choir was constructed first, being finished in 1327, while the construction of the entire nave continued until 1417. In 1422 work stopped, leaving the cimborio unfinished and a provisional wall closure as a façade. The building has a three-nave plan (the nave and two aisles) although, as a consequence of its particular design, it appears to enclose two additional aisles. This particular effect is caused by the inclusion of the imposing buttresses in the interior space between the side chapels. The nave spans 12.80 m and has a maximum high of 25.6 m. The span of the side aisles is equal to one half the span of the nave. The rise of the vaults at the side aisles, of 20.5 m, begins close to the springings of the central vaults. Thanks to this particular arrangement, the lateral thrust of the central vaults is efficiently carried to the buttress by the lateral vaults so that actual flying arches are in fact not needed. The overall system shows –as demonstrated by the analyses- large robustness thanks to the imposing buttresses (with a base of 7.4 m of length equal to 58% the maximum span) and the optimal structural arrangement. The existing flying arches are but draining devices with no structural role.

The robustness of the buttresses' dimensioning is easily apprehended when compared with other Cathedrals (see Table 1 in Appendix 2).

The analyses predicted that the structure does not experience significant damage when subject to dead load. Actually, no significant damage has been detected in the structure. On the other hand, further increases of load are allowed until causing the failure at 200% of the dead load (200% and 210% according to GMF and FEM-continuum damage model respectively). Although this value does not have a clear meaning (not allowing an identification as a truly "safety factor"), it gives idea of the resistant sufficiency of the construction.

Fig. 8 shows the distribution of damage in tension predicted by the FEM-continuous damage model for dead load and for a fictitious increased dead load leading to failure. Damaged zones in tension first appear at the crown of the transverse arches of the nave. As additional load is progressively applied, damage tends to cover larger regions of the structure. Further damage focuses are observed at the crown of the aisle vaults, at the haunches of the aisle arches and at the bases of the piers. Compression damage keeps almost null but for very high levels of load applied. As can be seen in Fig. 5, damage tends to concentrate in the regions where severe cracking also appears in the GMF model associated to the development of plastic hinges. The parametrical studies showed that a moderate decrease of the assumed compressive strength of the materials would not provide a significant variation of the ultimate capacity.

According to the analyses, the construction can resist a differential settlement of larger than 3 cm without experiencing severe damage. This illustrates the extreme ductility of the system.
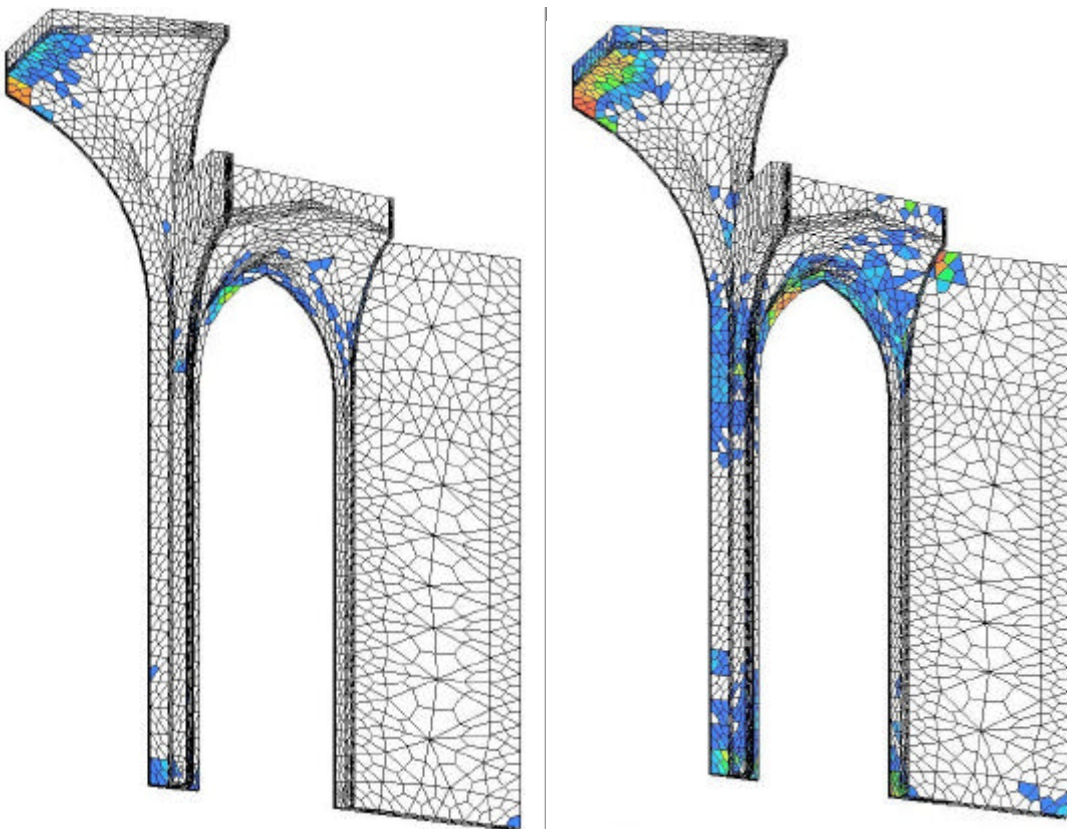
Figure 7: Nave of Barcelona Cathedral

Figure 8: Distribution of damage parameter in grey scale (between 0-intact and 1-full damage) for the structure of the nave of Barcelona Cathedral subject to dead load (left) and multiplied dead load leading to failure.

6. MALLORCA CATHEDRAL

6.1 Structural features

Mallorca Cathedral, begun in the 1350, is one of the most imposing medieval constructions thanks to the immensity of its interior space and the extraordinary dimensions and the extreme slenderness of its structural elements. Its 44 m vault keystone height is only exceeded by the choirs of Beauvais and Cologne cathedrals, while the free span of 17.8 m of its main arcade is only surpassed by the 21.8 m wide unique arcade of Girona Cathedral.

The main piers supporting the vaults and clerestory walls have octogonal section with diameter of 1.6 or 1.7 m. The slenderness of the piers, reaching a ratio of 13.8 between diameter and highness, constitutes the more unique and audacious aspect of the building and contributes largely to a sense of internal great spaciousness; in the case of other medieval cathedrals, this value stays between 8 and 9 (9.7 in for the piers of the choir of Beauvais Cathedral). The trend towards slenderness can be found in other structural components. The vaults, spanning 17.8 m, have a thickness of only 20 cm.

Because of these structural features, the building may easily suggest a sense of structural audacity to any analyst aware of most common proportions in Gothic or in general ancient masonry construction.

The diaphanousity of the interior space is made possible, in fact, by the very robust external buttressing system provided to the construction. The base of the main buttresses is 7.7 m long and 1.5 m wide; its maximum dimension represents a 44% of the span of the central nave. However significant these dimensions may seem, they are in fact closer to the proportions of Tarazona Cahtedral (which, as demonstrated, approach the limit of structural insufficiency) than to those of Barcelona Cathedral (showing reassured structural sufficiency).

In fact, the building is showing today a certain number of structural anomalies (large deformations and structural cracks) which significance is still subject to discussion. The main observed structural irregularities are:

1. Significant deformations affecting the piers, which show a remarkable curvature and lateral displacement both in the longitudinal and transverse direction of the nave.
2. Vertical cracks at the base of some of the piers; eventually, these cracks shape surface wedges partially expelled from the core of the pier (Fig. 12).
3. Significant deformations affecting the flying arches –in special, those corresponding to the upper battery. Apparently, a few flying arches were, at some time, propped by means of masonry columns and walls to prevent their possible failure (Fig. 10).
4. The vaults of the central nave and the main transverse arches are separated by wide cracks developed throughout their contact lines.

Cracking is also observed in other structural components (as in buttresses, caused by existing openings or false windows between lateral chapels, and also in lateral vaults).

Because of the concern caused by these observed anomalies –and, in particular, by the cracks and deformations affecting the piers of the central nave- a detailed assessment has been lay-out devising comprehensive historical investigation, inspection, monitoring and structural analysis (González and Roca 2000). While this action is scheduled for the immediate future, some preliminary considerations can be presented here with regard to the difficulties that may be encountered at the attempt to actually conclude about the condition of the structure.

Previous structural analyses are available thanks to the pioneering studies carried out by architect Josep Rubió (1912), consisting of a detailed static analysis, and Robert Mark (1984), by means of photo-elasticity. Additionally, preliminary analyses are also being carried out by means of the numerical techniques already mentioned (Continuous damage model and GMF). Comparing the results produced by these alternate approaches is of large interest because of the coincidences and, more important, because of the disagreements or contradictions they show.

Figure 9: Nave of Mallorca Cathedral

Figure 10: Double battery of flying arches

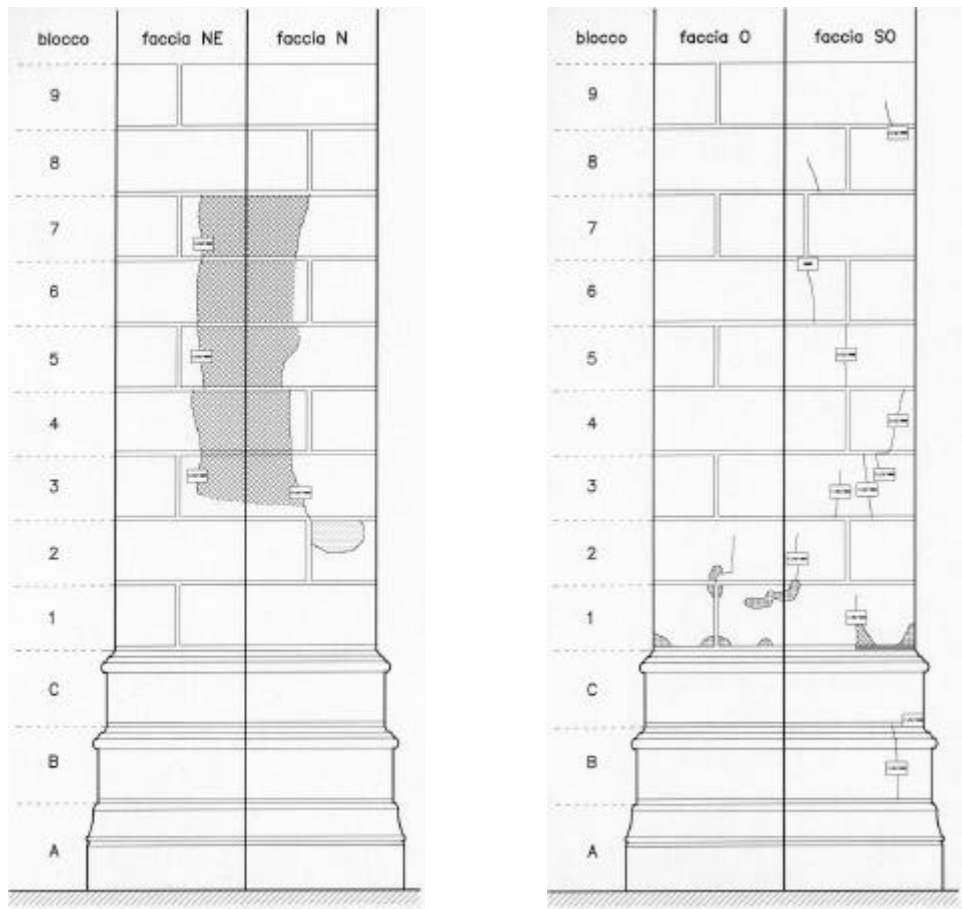Figure 11: Masonry dead weights placed on top of the arches and vaults of the nave

Figure 12: Elevation of cracks observed in two piers. In one of the piers (left) the cracks shape a volume of material partially detached.

6.2 Josep Rubio's study

Rubió's analysis, based on graphic- statics, was actually pioneering at its time. Although the concept was now, very few attempts had been carried before to apply it to a large and complex structure. Rubió succeed in applying it, in a very accurate way, to the case of Mallorca Cathedral.

After much elaboration, Rubió was able to find an equilibrated solution for which the thrust line kept fully contained within the volume of the elements. As explained by the author, fitting the descending thrust line within the volume of the pier revealed extremely difficult. In his solution, the thrust line becomes almost tangent to the perimeter of the pier at the level of the springing of the lateral vault (Fig. 13). Rubió noted that this solution was consistent with the curvature shown by the pier.

He estimated that the main arches would experience a maximum compression of 3.1 MPa, which should be considered rather high for the type of limestone masonry that compose them; the estimated maximum compression in the piers was 4.5 MPa.

Rubió was not fully satisfied with the solution obtained because it produced a very demanding, extreme condition in some of the structural elements and, in special, in the piers. However, all his attempts to find an alternate, less demanding form of equilibrium failed; this fact, still, does not mean that his solution is the only possible. As stated by himself, "the solution obtained, even if satisfactory, does not fully content the spirit nor it is unquestionable"
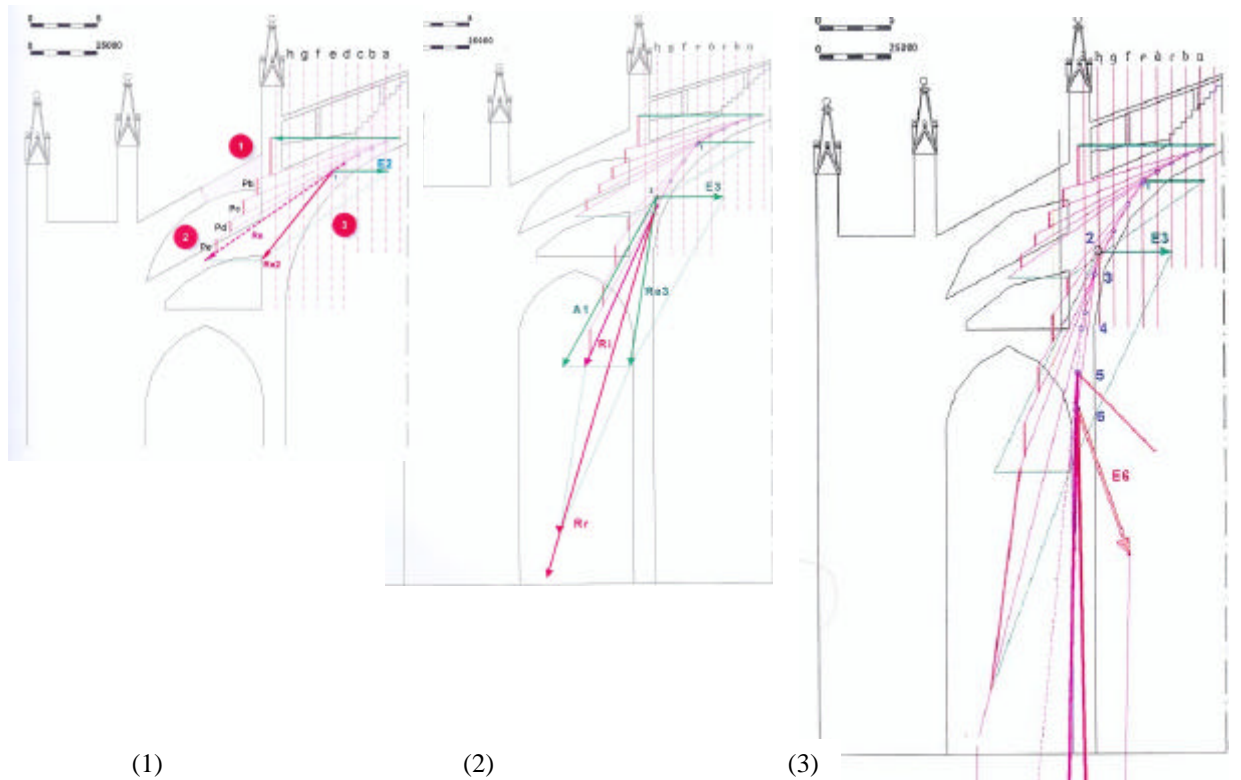
(1)   (2)   (3)

Figure 13: Three stages in Rubió's analysis: (1) The trhust of the main arch is evaluated and composed with the force applied by the upper flying arch; (2) the resultant is composed with the forces caused by the diagonal ribs of the vault and the lower flying arch; (3) the forces corresponding to the dead load of the system and the thrust of the lateral nave are added to obtain the resultant force at the base of the pier. A similar process is carried out to to compute the resulting force in the buttress.
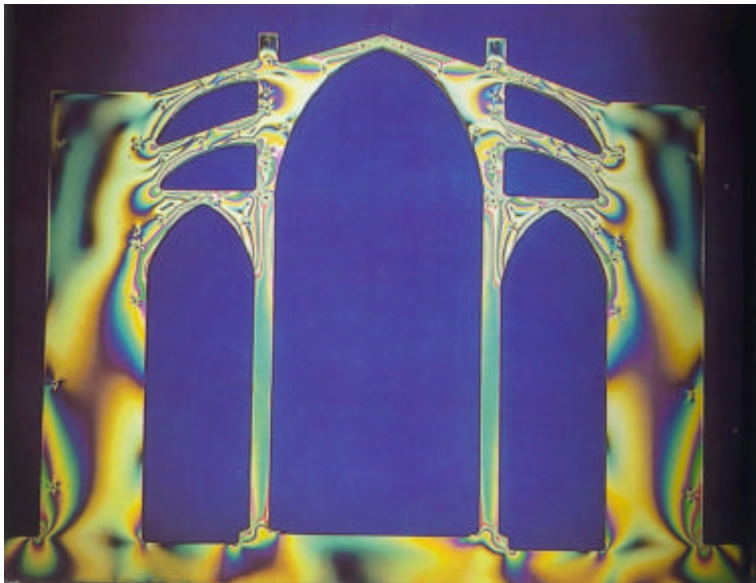

Figure 14: Photo-elastic modelling of the nave of Mallorca Cathedral by Mark (1984)

According to Rubió, attaining stability in the real construction should have required the inclusion of artifices such as the dead loads (in the form of masonry pyramids) placed over the vaults and main arches (Fig. 11); at least, his calculation showed that such contribution was required to obtain the equilibrium solution.

6.3 Robert Mark's study by photo-elasticity

The pioneering studies on the structure of Gothic Cathedrals carried out in the 70's by Robert Mark (1982) included the analysis by photo-elastic modelling of many emblematic Gothic constructions. Given its structural interest, the case of Mallorca Cathedral was also considered and analysed using the same technique.

The analysis permitted to draw interesting conclusions about its structural features and response subject to gravity loading and wind. Interestingly, some of the conclusions reached by Mark are not in agreement with those drawn by Rubió. According to Mark (1982), the photo-elastic study predicts a very uniform state of compression in the piers under dead-weight, indicating that the amount of bending is so negligible as to be unique among the many Gothic churches discussed by the author.

6.4 Historical research

A significant amount of historical information about Mallorca Cathedral is now becoming available thanks to the investigation carried out by Domenge (1997) and other specialists.

Remarkably, there is historical evidence about some partial collapses. According to the historical records, one of the arches of the nave collapsed in 1490 while some vaults had to be reconstructed during the 17th and 18th c. No much information is available on the extend of these collapses.

By March 1851 it was decided to dismantle and rebuilt part of the West façade because of the important and progressing plumb. In May, 1851, an earthquake occurred causing some damage to the façade. Basing on some historical records on the effects of the earthquake on people and buildings and, an intensity of VI-VII is estimated (in Mercalli's scale). During 1854 to 1861 works were finally undertaken to dismantle the upper part of the façade and to build a new, more robust one. The vaults of the bay close to the West façade were also dismantled and rebuilt during the process.

6.5 Present analyses

It is expected that the analyses carried out by means of the computer techniques mentioned (Appendix 1), now in course, allow additional insight on the features and structural performance of the building. In particular, the condition of the structure should be placed between the two opposites -the daring, extreme condition envisaged by Rubió, or Mark's more uniform, convenient state of forces.

Although the study is still in development, it is possible to advance some results (Fig. 15).

First, it has been verified that the behaviour of the construction is very sensitive to the ratio between the stiffnesses of the piers and the buttresses. In other words, the distinct stiffness of the vertical elements (piers and buttresses) has significant influence on the resulting distribution of stresses

When the stiffness of the material of piers and buttresses is set to a similar value, the solution obtained for the structure subject to dead loading is close to that obtained by Mark, i. e., mostly uniform and moderate states of compression are estimated for the piers and for the rest of the elements.

When the stiffness of the material of the piers is set to a value significantly larger than that of the buttresses, then larger eccentricities appear at some sections of the piers and, in overall, the results turn out more similar to the solution of Rubió.
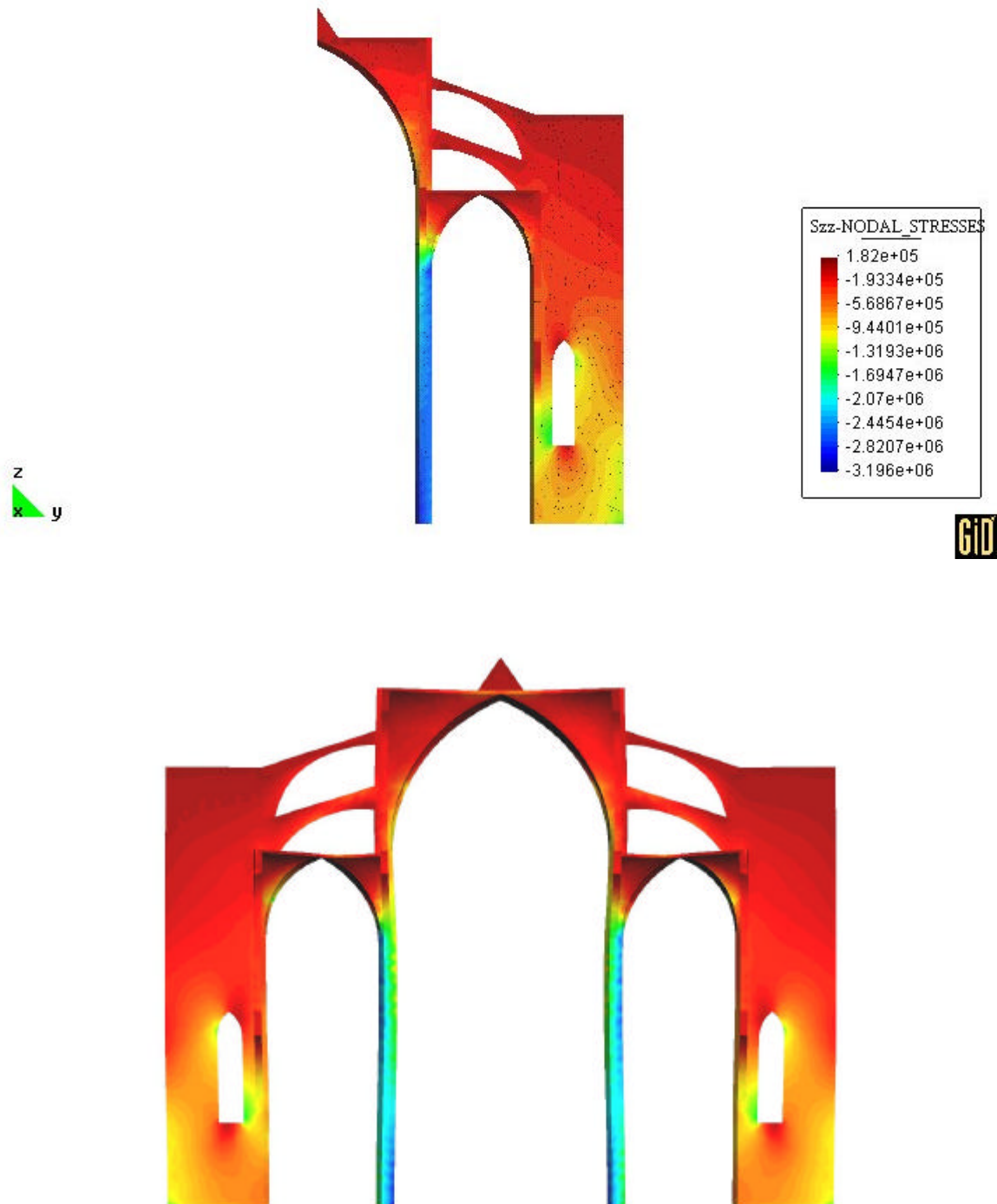
Figure 15: Distribution of the principal compression stresses for the cases in which the material of buttresses and piers is considered with the same stiffness (above) and the piers are considered significantly stiffer than the buttresses (below).

The stiffness of these elements is different because of their internal composition and the different materials used to build them. The pies are made of large stone block masonry of a very good quality sandstone; their section is almost fully supplied by the blocks with no significant amount of fill but for a very reduced core. The buttresses are made of a not-so resistant block masonry and contain a large amount of rubble infill. Because of that, the stiffness of these elements may be actually very different.

Second, the deformations of the structure have been investigated in the light of the numerical predictions. For that purpose, the structure has been studied subject to gravity by means of non-linear geometric and material analysis. In turns out that, for gravity forces, a good approach of the actual deformed shape is obtained but only qualitatively (Fig. 15). In absolute figures, the deformations predicted by the numerical models (of the order of mm) do not approach at all the large deformation actually existing in the piers (of the order of tens of cm). The values of the real deformations cannot be explained because of the unique competition of the gravitational action, even in the context of a non lineal analysis. Certainly, the deformations of the piers may have been amplified by non-mechanical effects such as the plastic settlement of mortar at an early age or the long-term creep of mortar and stone; but even if these contributions are considered, the actual existing deformations seem too larger in magnitude to be explained as a consequence of gravity only.

In our opinion, the large existing deformations can only be explained by historical processes developed in the term of several centuries. Besides the possible creep, it seems important, even much more important, the effect of the thermal cycles experienced in such a long term. A very small remaining increment produced after each cycle might well justify, for accumulation, the current deformational condition. Earthquakes and micro-tremors may have contributed, in a more punctuated way, to also amplify the deformation.

The existence of vertical cracks at the base of the piers is also difficult to justify based on the prediction of the numerical models for the structure subject to gravity. The fact that the cracks are mostly vertical and develop through both the joints and the blocks suggests that they are caused by excessive compression, that is to say, by significant load eccentricity at the base of the pier. However, excessive compression leading to vertical cracking can only be justified numerically if a very reduced, perhaps unrealistically low value of the compressive strength of the masonry of the piers is assumed.

Again, understanding the cause of such cracks may require the contribution of historical or long-term actions such as earthquakes, hurricane wind, long-term fatigue and other.

Construction or utilisation practices may be also involved in the production of the cracks. On one hand, wooden wedges were used to support the blocks during the erection of the piers, which were left embedded in them, and may have caused a significant concentration of stresses. On the other hand, the insertion of iron nails in the mortar joints or even in the blocks, to sustain ornamental or liturgical devices, may have acted similarly causing concentration of stresses and possible cracking.

6.6 On the role and deformation of the flying-arches

The work of the two authors above mentioned (Rubió 1912, Mark 1984) suggests that the second, higher battery of flying arches of the building lacks evident structural role and even causes some undesired resistant effects which, following Rubió, require additional artifices (such as the dead weights over the arches and vaults) to compensate for them. The inclusion of this upper battery could thus be observed as a flaw in the structural understanding of the ancient constructors who, apparently, transposed to Mallorca Cathedral a design of the largest High Gothic cathedrals (as in Amiens, Beauvais and Cologne) which, in fact, was unsuitable for it. Upper flying arches would only reach full structural sense if vaults had been covered with a high pitched roof, which has never been the case of Mallorca Cathedral.

However, an alternate non-structural explanation can be forwarded to understand why the upper battery of flying arches was built. This alternate explanation comes from the need to drain the large roof of the central nave, covering more than 1200 m^2. Mallorca Cathedral was not

originally provided with a high pitch roof but with a flat, tiled one, like most Gothic Cathedrals built in the Kingdom of Aragón. It must be said that although the Mediterranean climate can be considered dry in terms of average pluviometry, rain can manifest with intensity during short periods thus demanding significant drainage capacity. Thus, the upper flying arches were probably built mostly as draining channels springing from the natural draining, low points of the vaulted roof. Since they had to span more than 8 m, these arches needed to be robust and were built similar to the fully structural, lower ones. However, in adopting that solution, secondary mechanical effects were induced. These generated effects disrupted the equilibrium and required complementary balancing devices such as the masonry pyramids placed on top of arches and vaults (as already mentioned).

The second paradox posed by the flying arches comes from their remarkable deformed shape. As already said, most of the flying arches of the building show today a very a large deflection which can be recognised thanks to the curvature acquired by the cornice (Fig. 10). This deformation may have created deep concern at certain (historical) time and led to propping some of the upper flying arches on masonry columns built over the lower ones. However, the possibility of the deformed shape of the flying arches being a construction treat –that is to say, a consequence of a simplified or pragmatic construction process- should be carefully assessed.

A larger discussion on the hypothesis mentioned with regard to the role and deformed geometry of the flying arches of Mallorca Cathedral can be found in Roca and González (2001).

7 FINAL REMARKS

The three different case studies here presented illustrate the very diverse conditions that may be experienced by a Gothic construction due to its particular structural design and its sensitivity to historical actions. The state of the cases presented range from severe structural disorder (Tarazona Cathedral) to almost intact structural integrity (Barcelona Cathedral). Eventually, the case of an audacious structure (Mallorca Cathedral) may challenge the capacity of the analysts to conclude on the adequacy of the design and the actual meaning of its deformations and lesions.

The cases presented also show the need for an integrated approach comprehending structural and historical analyses. On one hand, the history (including the present reality) of the building must be accepted, by all means, as the main source of evidence on its structural performance and present stability condition.

On the other hand, interpreting history and present reality requires some form of structural analysis, based on numerical and experimental techniques. The need for the structural analysis comes from the wish for objectivity and quantification.

In the case of Tarazona, the structural analysis provided quantitative understanding to a historical fact, namely the structural insufficiency of the construction. In the case of Barcelona Cathedral, the analysis provided a rational understanding of a foreseeable fact –its structural robustness.

The case of Mallorca Cathedral is more challenging because of the daring structural features of the building and the difficulties encountered in the attempt to interpret the cause and significance of the existing deformations and lesions. A campaign including thorough historical research, structural analysis and monitoring is due in the years ahead. Some previous calculations, based on different approaches, have produced results that are, to a certain extend, contradictory. Even the structural analysis currently carried out using sophisticate numerical tools shows to be insufficient to draw convincing conclusions.

The challenges posed by the structure of Mallorca Cathedral stem from the difficulty to gather and interpret complex reality and history. Any attempt to actually understand its structural condition and assess its long-term stability should be based on a sheerly integrated analysis combining structural evaluation, historical research and monitoring.

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APPENDIX 1 – METHODS OF ANALYSIS USED FOR THE PRESENT STUDIES

Non-linear formulation for 3D framed structures with curved members (GMF). This approach consists of the modelling of the ancient structures of Gothic Cathedrals as an equivalent frame with one-dimensional spatial curved elements. These elements are used to describe the piers, abutments, flying arches and vault ribs. For that purpose, a flexibility formulation for 3D framed masonry structures with curved members, based on a generalisation of conventional matrix methods, has been adopted (Molins and Roca 1998).

Consistently with the principles of matrix methods, the flexibility formulation stems exclusively from equilibrium between external and internal forces at any point within an arch or linear member, so that no additional hypotheses over the displacement or stress field are required. Since the movements are fully free (unlike in FEM, where field displacement shapes must be assumed), arbitrarily high concentrated curvatures associated with damage can be reproduced, resulting in a feasible approach for damage localisation, or hinge formation. In order to carry out the non-linear material analysis, masonry is treated as a linear elastic-perfectly brittle material under tension, while elasto-plastic equations are adopted for masonry subject to compression and shear. A Mohr-Coulomb failure envelope is adopted in order to describe failure modes due to combined states of compression and shear.

In order to carry out the analysis using the methods referred to in Appendix 1, a model was elaborated representing the typical bay of the nave, including the piers, buttresses, flying arches and vaults, all modelled by means of straight or curved linear elements. The vaults of the nave and those of the aisles are modelled as arches with a complex, variable cross section incorporating the transverse rib, part of the membrane of the cross-vault and the masonry backing which exists over the springings of the vault. According to the experience of the authors, this treatment can give acceptable results about the equilibrium and strength of the structure in spite of the coarse approach used to model the vaults.

3D Finite Element Damage Continuum Model. Continuum damage models are particularly useful for the simulation of fragile materials such as concrete, ceramics and stone.   In this work two formulations recently developed by Cervera et al. (1998) and Oñate et al (1997) for non-linear analysis of concrete, based on the concepts of *damage* above mentioned, have been chosen. These formulations are based on a isotropic damage model with only two internal scalar damage variables to respectively characterise tension and compression damage. This yields a simple constitutive equation which nevertheless enables to simulate all the important aspects of the non-linear behaviour of concrete and masonry, such as the different response under tension and compression, softening due to deformation, and the stiffness degradation due to compression-tension cycles.   The damage variables can take values ranging from 0, for undamaged intact material, to 1 (in fact an unreachable bound), for the complete loss of resistance at micro-structural level. The loss of stiffness at each material point is then assumed to be proportional to the damaged parameter, which evolution from zero to one is adequately characterised by an experimental law defined via experimental testing.

APPENDIX 2- COMPARISON OF GEOMETRIC PARAMETERS

Table 1 – Main dimensions and geometric proportions in Tarazona, Barcelona and Mallorca Cathedrals

	Tarazona (initial)	Tarazona (later)[1]	Barcelona	Mallorca
Highness of the nave (m)	17.8		25.6	44
Span of the nave (m)	6.7		12.8	17.8
Average compression estimated at the base of the pier (MPa)	2.1		3.0	2.6
Main dimension of buttress / span of the nave	0.36	0.46	0.58	0.44
Dimension of pier / dimension of the buttress	0.55	0.43	0.36	0.19
Mass of buttressing system / Total structural mass (for a typical bay)	0.04	0.05	0.10	0.08
Pier slenderness Free highness[2] / diameter Total highness[3] / diamete[3]	3.1 9.1		9.8 9.8	14.1 19.4

[1]In the North side, after the enlargement of the section of the buttresses
[2]up to the springing of the lateral nave
[3]up the springing of the central nave